GAUGE INVARIANT PROPERTIES OF ABELIAN MONOPOLES

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Using a renormalization group motivated smoothing technique, we investigate the large scale structure of lattice configurations at finite temperature, concentrating on Abelian monopoles identified in the maximally Abelian, the Laplacian Abelian, and the Polyakov gauge. Monopoles are mostly found in regions of large action and topological charge, rather independent of the gauge chosen to detect them. Gauge invariant properties around Abelian monopoles, the local non-Abelian action and topological density, are studied. We show that the local averages of these densities along the monopole trajectories are clearly above the background, which supports the existence of monopoles as physical objects. Characteristic changes of the vacuum structure at the deconfinement transition can be attributed to the corresponding Abelian monopoles, to an extent that depends on the gauge chosen for Abelian projection. All three Abelian projections reproduce the full $SU(2)$ string tension within 10 % which is preserved by smoothing.

1 Introduction

Over the last two decades a variety of attempts in field theory have been aiming for a qualitative understanding and modeling of two basic properties of QCD: quark confinement and chiral symmetry breaking. The most prominent schemes are the instanton liquid model [1] and the dual superconductor picture of the QCD vacuum.[2] While the first model explains chiral symmetry breaking and solves the $U_A(1)$ problem, the second one provides a simple idea for the confinement mechanism. In this scenario, where the vacuum is viewed as a dual superconductor, condensation of color magnetic monopoles leads to confinement of color charges through a dual Meissner effect. The superconductor picture was substantiated by a large number of lattice simulations over the last years. So it was shown that in the confinement phase monopoles percolate through the $4D$ volume [3] and are responsible for the dominant contribution to the string tension.[4] At present, more and more groups characterize their lattice vacuum in accordance to the instanton liquid picture.[5]

Both models rest on the existence of very different kinds of topological excitations, instantons and color magnetic monopoles. For a long time they have been treated independently, only recently some deeper connection among those different objects has been pointed out, both on the lattice and in the continuum.[6] Instantons are localized solutions of the Euclidean equations of motion in Yang-Mills theory carrying action and integer topological charge. Even though it is difficult to detect instantons and antiinstantons among quantum fluctuations, there is no problem to study these well-defined objects in classical or semiclassical (heated) configurations on the lattice. The situation for monopoles is more difficult. Following 't Hooft, monopoles should be searched for as pointlike singularities of some gauge transformation dictated by a local, gauge covariant composite field. The standard prescription, however, is localizing monopoles in QCD as Abelian monopoles via an Abelian projection from some gauge (for example the maximally Abelian gauge). This leads to monopole trajectories which are dependent on the gauge chosen. Note however, that the condensation mechanism of monopoles itself seems to be gauge independent.[7]
In this contribution we relate gauge invariant observables to monopole trajectories, with the intention to further understand the semiclassical vacuum structure in terms of monopoles and lumps of topological charge and their role for the confinement problem. We will comment on a new way of monopole identification on the lattice which evades serious problems of previous methods, and which might have a close formal relationship to 't Hooft-Polyakov monopoles.

2 Smoothing

To resolve semiclassical structures in gauge field configurations provided by lattice simulations, the cooling method has been used, which locally minimizes the action. However, even improved versions of cooling rapidly destroy monopole percolation and reduce the string tension. From the instanton point of view cooling is known to destroy small instantons and instanton-antiinstantons pairs, such that the true topological structure is acces-

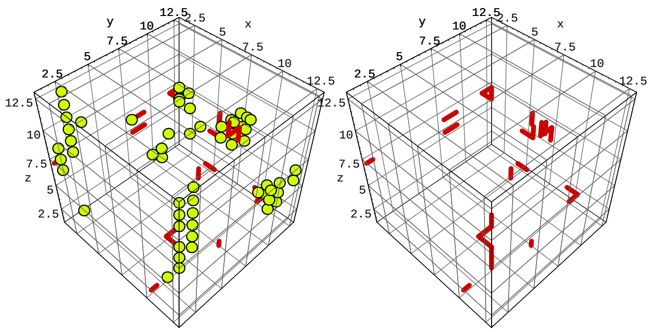

Figure 1: Regions of low modulus of the auxiliary Higgs field (dots), which *should* mark the trajectories of monopoles according to LAG, are found very close to the trajectories of DGT monopoles obtained by Abelian projection. For clarity DGT monopoles are also shown alone (right).

sible at best by a backward extrapolation to zero cooling steps. Up to now, most lattice studies are performed with the Wilson action, for which the lattice definitions of the topological charge Q are known to violate the bound valid for the continuum action: $S \geq 8\pi^2|Q|/g^2$, where g is the coupling constant. The Wilson action is known to decrease with smaller size ρ of an instanton, such that isolated instantons are unstable under cooling. In contrast to this, improved cooling finds instantons stabilized within a size interval $\rho > 2\ a$ (a is the lattice spacing). Other methods like APE smearing let instantons grow.

To avoid these ambiguities we have used a method of 'constrained smoothing' [8] which is based on the concept of perfect actions.[9] These actions respect the above bound for the topological charge and lead to a theoretically consistent 'inverse blocking' operation. Inverse blocking is a method to find a smooth interpolating field on a fine lattice by constrained minimization of the perfect action, provided a configuration is given on a coarse lattice. This makes an unambiguous definition of topological charge possible. Constrained smoothing is a renormalization group motivated method which first blocks fields $\{U\}$, sampled on a fine lattice with lattice spacing a, to a coarse lattice $\{V\}$ with lattice spacing $2\ a$ by a standard blockspin transformation. Then inverse blocking is used to find a smoothed field $\{U^{\rm sm}\}$ replacing $\{U\}$.

An important feature of this method is that it does not drive configurations into classical fields as unconstrained minimization of the action would do. It saves the long range structure of the Monte Carlo configuration in $\{V\}$, such that the smooth background contains semiclassical objects deformed by classical and quantum interaction. The upper blocking scale roughly defines the border line between 'long and short range'[a]. In this work we used a simplified fixed-point action [11] for Monte Carlo sampling and for constrained smoothing before the configurations were analyzed.

[a]The iterative application of this method, 'cycling',[10] obscures the idea of a definite blocking scale while it still preserves rather well features of long range physics as the string tension.

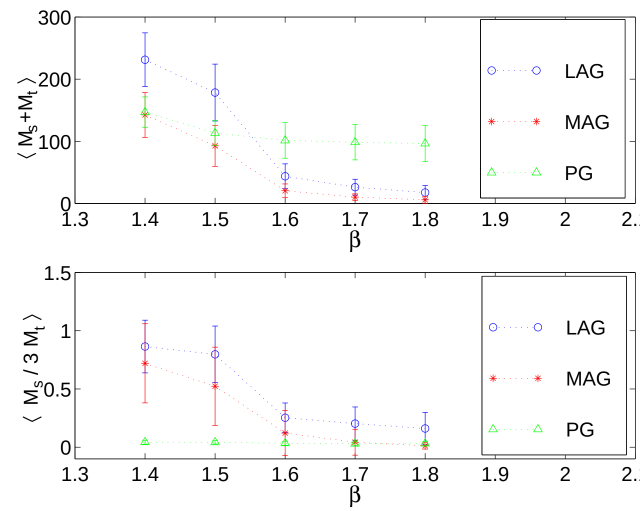

Figure 2: Total monopole length (top) and space-time asymmetry (bottom) as a function of β for monopoles obtained in different gauges. $\beta_c = 1.545(10)$ is the deconfinement point.

3 Gauge Fixing

The most popular gauge to study monopoles on the lattice is the maximally Abelian gauge (MAG).[12] This gauge is enforced by an iterative minimization procedure, which can get stuck in local minima, so-called *technical* Gribov copies. The Laplacian Abelian gauge (LAG) [13] is not afflicted by this problem. MAG and LAG can be understood along the same lines. The gauge functional of the MAG can be written:

$$F(\Omega) = \sum_{x,\mu}(1 - \frac{1}{2}\,tr\,(\sigma_3 U^{(\Omega)}_{x,\mu}\sigma_3 U^{(\Omega)\dagger}_{x,\mu})) \\ = \sum_{x,\mu,a}(X^a_x - \sum_b R^{a,b}_{x,\mu}X^b_{x+\hat\mu})^2 \rightarrow \int_V (D_\mu X)^2\,, \quad (1)$$

with the gauge transformation Ω_x acting on $\{U\}$

$$U^{(\Omega)}_{x,\mu} = \Omega_x U_{x,\mu}\Omega^\dagger_{x+\hat\mu}$$

encoded in an *auxiliary* adjoint Higgs field

$$\Phi_x = \Omega^\dagger_x\sigma_3\Omega_x = \sum_a X^a_x\sigma_a$$

subject to local constraints $\sum_a (X^a_x)^2 = 1$ and with adjoint links

$$R^{a,b}_{x,\mu} = \frac{1}{2}\,tr\,(\sigma_a U_{x,\mu}\sigma_b U^\dagger_{x,\mu})\,.$$

In LAG the local constraints are relaxed and replaced by a global normalization: $\sum_{x,a}(X_x^a)^2 = V$, such that Eq. (1) can be further written:

$$\int_V (D_\mu X)^2 \to \sum_{x,a}\sum_{y,b} X_x^a\{-\Box_{x,y}^{a,b}(R)\}X_y^b\,. \qquad (2)$$

Then the minimization reduces to a search for the lowest eigenmode of the covariant lattice Laplacian. LAG is unambiguously defined, except for degenerate lowest eigenmodes, which correspond to *true* Gribov copies. For both MAG and LAG, the gauge transformation is finally performed by diagonalization of the field Φ_x. Quite similarly we enforce the Polyakov gauge (PG) by diagonalization of Polyakov loops.

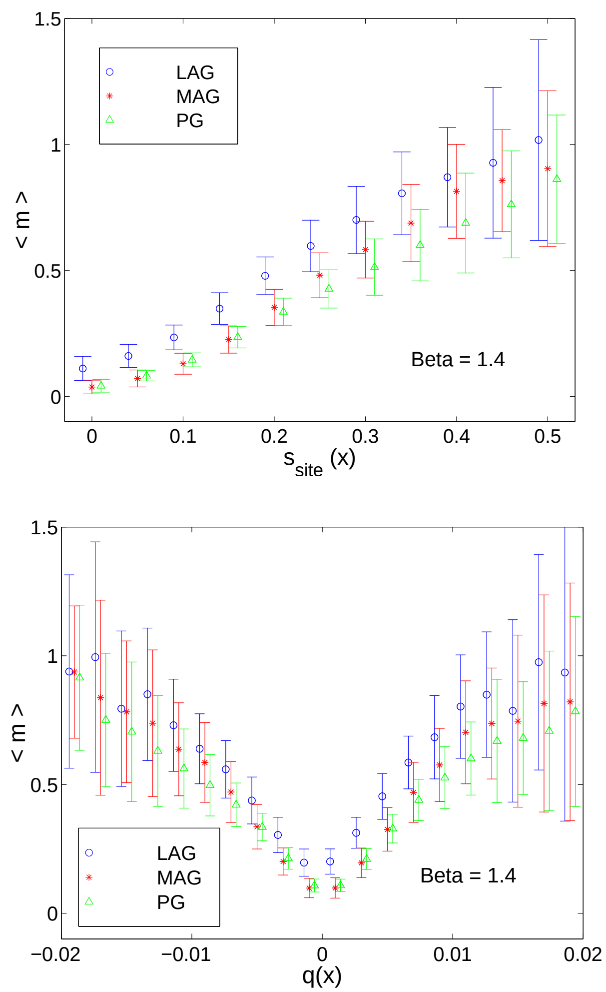

Figure 3: Average occupation number of monopoles $< m >$ nearest to sites with action density $s_{\rm site}$ (top) and topological charge density q (bottom) in the confinement phase.

After the Abelian gauge of choice has been fixed one extracts the Abelian degrees of freedom (Abelian projection). The Abelian link angles can then be used for the identification of monopoles, like in compact $U(1)$ theory, searching for the ends of Dirac strings. Monopoles identified in this manner are generally referred to as DeGrand-Toussaint (DGT) monopoles. The Higgs field indroduced in the LAG provides an alternative for monopole identification which is more satisfactory from a physical point of view. Lines of $\rho_x = |\Phi_x| = 0$ where ρ_x is defined as

$$X_x^a = \rho_x \hat{X}_x^a \quad , \quad \rho_x = \sqrt{\sum_{a=1}^{3}(X_x^a)^2}\,, \qquad (3)$$

directly define lines of gauge fixing singularities (monopoles), more in the original spirit of 't Hooft.[b] Note here that this way of monopole identification does not require to perform the actual gauge fixing and Abelian projection!

In Fig. 1 we show that both methods of monopole identification turn out to be quite related. Regions of small ρ are highly correlated with trajectories of monopoles identified by the DGT method.

4 Physical Properties of Monopoles

The following results were obtained from simulations of pure $SU(2)$ theory on a $12^3 \times 4$ lattice. Observables were computed on 50 independent configurations per β. Different β values were considered to study the behavior slightly below and above the deconfinement phase transition. For the particular action used,[11] $\beta_c = 1.545(10)$. Global properties like the total loop length and the space-time asymmetry are shown in Fig. 2. DGT monopoles extracted from the MAG and the LAG behave qualitatively similar. Those from the the PG show no change at the deconfinement phase transition. This reflects the fact that PG monopoles should be static.

In Fig. 3 we present the average occupation number of monopoles on dual links nearest to a given site as a function of the local action $s_{site}(x)$ and charge $q(x)$, for different gauges. One observes that the probability of finding monopoles increases with the amount of action/charge density at the same lattice position. This result is practically independent of the gauge used to define the (DGT) monopoles.

If monopoles are physical objects, one expects that they can be characterized by a local excess of the (gauge invariant) action. We define such an excess action of monopoles by

$$S_{\rm ex} = \frac{< S_{\rm monopole} - S_{\rm nomonopole} >}{< S_{\rm nomonopole} >}\,, \qquad (4)$$

[b]For the 't Hooft-Polyakov monopole regions with $\rho = 0$ of the *physical* Higgs field are identified with the centers of such monopoles.

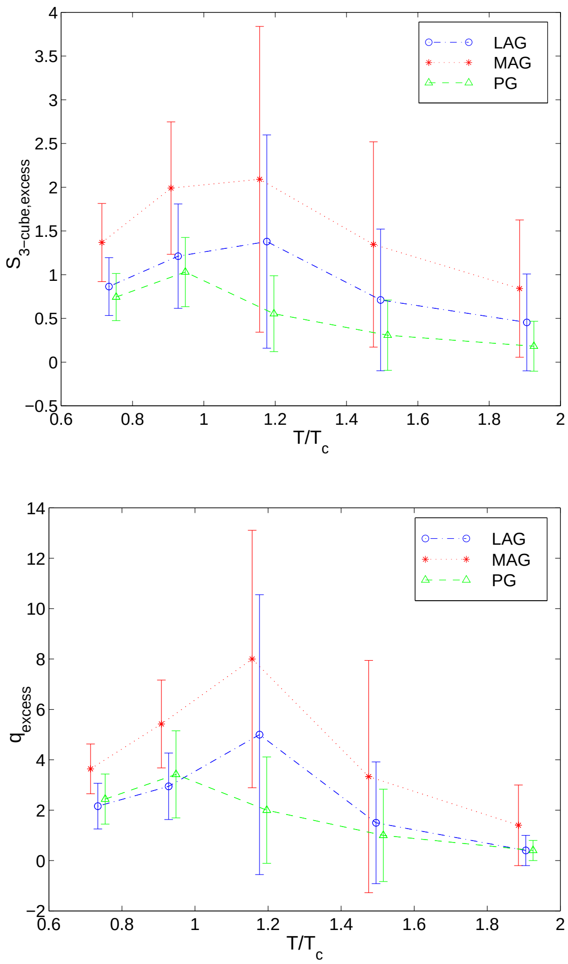

Figure 4: Excess action (top) and charge (bottom) as a function of temperature.

where $S_{\rm monopole}$ is the action contained in a three-dimensional cube which corresponds to the dual link occupied by a monopole. Replacing the action in the above expression by the modulus of the topological charge density according to the Lüscher method we obtain the charge excess $q_{\rm ex}$. For details of the definition of the local operators see Ref. 11. Fig. 4 shows that just below T_c the excess action and charge for the MAG and LAG monopoles are clearly above one, indicating an excess of action of more than a factor of two compared to the bulk average (background). The large error bars above T_c reflect the fact that the topological activity diminishes in the deconfinement phase. These results are somewhat enhanced in comparison to a $T = 0$ study with Wilson action without cooling or smoothing.[14]

In Fig. 5 we display static quark-antiquark potentials obtained from Polyakov-Antipolyakov correlators, for the $SU(2)$ fields and, in the case of LAG and MAG, after Abelian projection. The Abelian string tension of

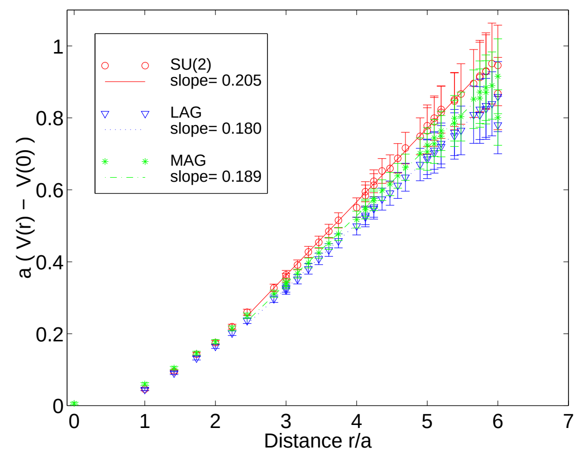

Figure 5: Static quark-antiquark potentials obtained from Polyakov-line correlators after smoothing. The slope of the Abelian potential in the MAG after Abelian projection is about 5% less than that of $SU(2)$ gauge field. Another 5% are lost in the LAG. Still the LAG carries 90% of the original string tension, indicating Abelian dominance for the LAG.

the MAG is about 5% less than that the $SU(2)$ field. The Abelian string tension of LAG is a little smaller than for the MAG but still exhibits Abelian dominance. The Abelian string tension of PG is trivially identical with that measured on the smoothed $SU(2)$ configurations.

Finally we present an intuitive argument, that monopoles also should carry electric charge, that they are dyons. Consider the selfduality equations $F_{\mu\nu} = \pm\tilde{F}_{\mu\nu}$. From the trivial relation

$$\int d^4x \mathrm{Tr}[(F_{\mu\nu} \pm \tilde{F}_{\mu\nu})^2] \geq 0 \qquad (5)$$

it immediately follows that

$$S \geq \frac{8\pi}{g^2}|Q| \qquad (6)$$

and (anti)selfdual fields saturate the identity. Fig. 6 depicts the probability distribution of topological charge density for a given local action and shows that for $s_{site} > 0.3$ the local action obeys a local version of Eq. (6) near to saturation, $s_{site}(x) \sim \frac{8\pi}{g^2}|q(x)|$. The plot was obtained after one constrained smoothing step, and exhibits that the gauge fields are already sufficiently smooth to expose semi-classical structure. This is suggested by the relatively clear ridges indicating approximate local selfduality for large enough action density. In Fig. 3 we provided evidence that monopoles are found predominantly in regions of large action. We thus conclude that monopoles also carry electric charge and should be interpreted as dyons. Note that this way of argumentation is a short-cut, to be more precise, one would have to test for local selfdualtiy along individual monopole trajectories.

Beta = 1.40

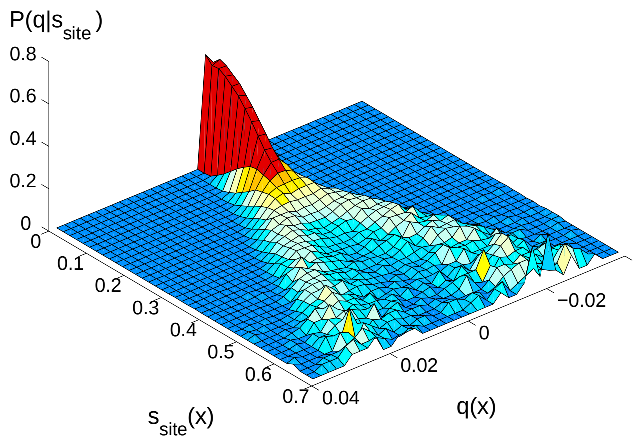

Figure 6: Probability distribution for finding a topological charge density $q(x)$ at a lattice site x if the local action density equals $s_{site}(x)$. The ridges follow the lines $s_{site}(x) = \frac{8\pi}{g^2}|q(x)|$ where local (anti)selfduality is satisfied.

5 Conclusions

We have demonstrated that the renormalization group smoothing technique with an (approximate) classically perfect action provides a powerful tool to investigate the semiclassical vacuum structure. Analyzing trajectories of monopoles identified in various gauges we found that monopoles appear preferably in regions which are characterized by enhanced action and topological charge density. We showed that in exactly those regions local (anti)selfduality of the gauge fields is prevailing. This is further evidence that monopoles should be addressed as dyons. We demonstrated that almost the complete string tension can be recovered from the Abelian projected field corresponding to various Abelian gauges, indicating Abelian dominance also for the LAG. This is trivially true for the PG, but the corresponding monopoles do not change at the deconfinement transition. We have shown that monopole trajectories carry an excess action of about twice the background action density of smoothed gauge fields. Similarly, monopoles also carry excess topological charge. In the confinement phase this observation is rather independent of the gauge chosen for identifying Abelian monopoles, but the behavior of PG monopoles is different in the deconfinement. We therefore conclude that MAG and LAG monopoles behave similar physically and can be interpreted as physical objects which carry action and topological charge.

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